Experimental Study of Hydraulic Jumps in an Inclined Rectangular Flume

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ABSTRACT

Hydraulic jumps on an inclined rectangular channel are the subject matter of this paper. The hydraulic jump is the sudden transition from a high velocity super critical flow to a subcritical flow regime in an open channel flow. The flow properties were solved using continuity and momentum principles. Laboratory experiments of hydraulic jumps in an inclined flume were conducted to verify the theoretical sequent-depth ratio, roller length and jump length. Measurements of velocities were made with an acoustic Doppler velocity meter for various Froude numbers flows. In the experiment, the bed slopes of 0.038, 0.094, 0.151, 0.210, 0.270, and 0.333 were used for measuring various flow parameters along the contour of the hydraulic jump. Results have shown that the sequent depth ratio increases with increasing the positive slope. Based on the present investigation the dimensionless length of the jump is significantly dependent on the bed.

Keywords: Bed slope, energy loss, hydraulic jump, inclined rectangular flume, sequent depth ratio

INTRODUCTION

An abrupt change from supercritical to subcritical flow is defined as a hydraulic jump. During the conversion energy is dissipated. With high turbulence, the potential break correlated with the jump has been acknowledged as the efficient method for indulgence of energy at the downstream of hydraulic structures. Hydraulic structures need active characteristics of hydraulic jump, equivalent to specific energies. The efficiency of the jump depends upon discharge Q and energy loss E_L of the jump. An assortment of methods and empirical equations are used to validate the characteristics of flow within the specified range. Barrage/weir is

designed with the help of analytical equations which are valid for all probable values of discharge and energy dissipation of the jump. Tail water depth and conjugate depth vary at different discharges.

The earliest experiments on the hydraulic jump, made by Bidone, were actually done in a sloping channel (Rajaratnam, 1967). Bazin in 1865 and Beebe and Reigel in 1917 also experimented on sloping channel jumps (Rajaratnam, 1967), and in 1927 Ellms attempted a theoretical and experimental study of this problem (Rajaratnam, 1967). In 1934, Yarnell started an extensive research program with slopes of 1 in 6, 1 in 3 and 1 in 1, which was unfortunately interrupted by his death in 1937 (Rajaratnam,1967). In 1935 Rindlaub conducted an experimental study of slopes at 8.20, 12.50, 24.20 and 300 (with the horizontal). Hager and Bremen (1990) had analysed the consequences of wall abrasion linking to sequent depth ratio. Ohtsu and Yasuda (1991) had resolved the issues related to D-jump and B-jump. Adam et al. (1993) had introduced new expressions for B-jump with Froude numbers of 2.4 to 7.4. Gunal and Narayanan (1996) had determined the variations of the mean stream flow with hydraulic jump in sloping channels. Ead and Rajaratnam (2002) had observed the range of Froude numbers from 4 to 10. Ohtsu et al (2003) had investigated undular jumps for completely built up inflow condition. Beirami and Chamani (2006) reported negative bed slope of the flow bed reduced the sequent depth ratio, and positive bed slope increased the sequent depth ratio. Carollo et al. (2007) studied effect of bed roughness on the sequent depth ratio and the roller length. Chanson (2009) had developed the recent advances in turbulent hydraulic jumps. Afzal et al., (2011) had investigated the stream flow of a turbulent hydraulic jump in a rough rectangular channel bed. Carollo et al. (2012) had studied the characteristics of classical jump and B-jumps on smooth beds. Wang and Chanson (2015) had conducted the experiments to study the fluctuation of Froude numbers ($3.8 < F1 < 8.5$) and Reynold's number ($2.1 \times 104 < R < 1.6 \times 105$). The main objective of this study is to present an advanced assessment of the comparative studies of hydraulic properties in terms of jump parameters with variation of bed slopes, and to compare the measured versus calculated parameters associated with hydraulic jump.
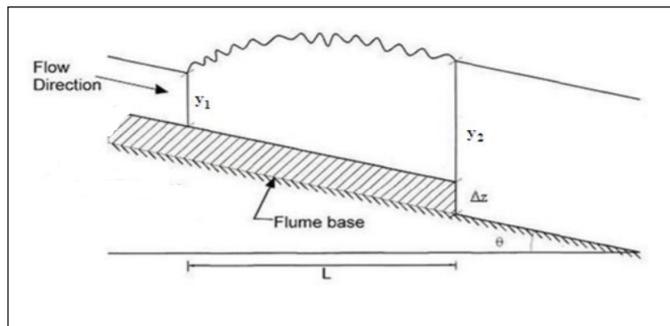
METHOD

The experiment was conducted on a tilting rectangular flume of 4.90 m long, 0.308 m width and 0.50 m deep, made of stainless steel frame with smooth bed. Side walls were transparent glass sheet for visualisation of flow represented in Figures 1 and 2.

Figure 1. Photograph of Experimental setup
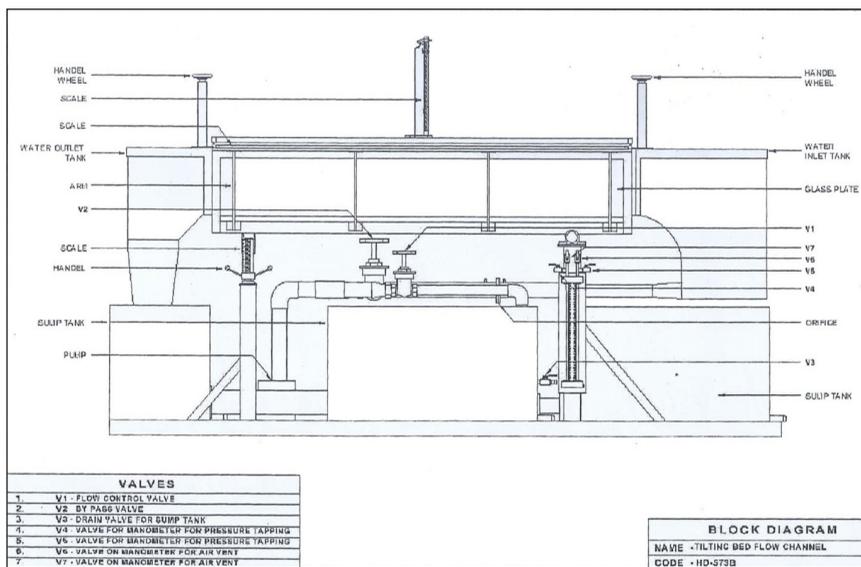
(a)

(b)

Figure 2. (a) Cross sectional view of experimental Tilting Flume; and (b) Schematic view of hydraulic jump (E-jump)

Experimental Procedure

The water was supplied from an inlet constant head tank through a sluice gate and discharges into water outlet tank. By controlling the tail water depth with adjustable downstream gate, the hydraulic jump was to be formed. A screw jack was located at 0.36 m distance from the downstream gate to obtain the slope of the channel. Six different bed angles 2.22°, 5.41°, 8.62°, 11.86°, 15.13°, and 18.46° respectively, were used for the laboratory experiments. Entry of water into the flume is measured with manometer. Constant head tank is used to measure the capacity of discharge. Discharge (Q), the upstream flow depth (Y_1), the downstream conjugate depth (Y_2), roller length (L_r), the horizontal distance between two section of the jump (L_j) were measured for each Hydraulic jump.

The following assumptions are considered for developing the relation between sequent depth ratio, upstream Froude number, and slope of the channel. The assumptions are: The channel is sloping, rectangular and straight. The pressure distribution is hydrostatic at both ends of the flume. Effect of turbulence is not considered during the study. The frictional resistance of sidewalls and bed of the flume are neglected.

Sequent Depth Ratio

Initial depth Y_1 is before the jump and the sequent depth Y_2 is after the jump. The sequent-depth ratio for a horizontal rectangular channel is given by the Belenger equation-

$$\frac{Y_1}{Y_2} = \frac{1}{2}(\sqrt{1 + 8F_2^2} - 1)$$

(1)

In an inclined channel the analysis of a hydraulic jump is done by considering the weight of water in the jump. For E-Jump the sequent depth ratio is calculated with modified Froude number by G replacing the Froude number F_1 as

$$G = \frac{F_1}{\sqrt{\cos\theta - KL_r \tan\theta / (Y_2 - Y_1)}}$$

(2)

L_r = roller length of the jump and; K = correction factor for the volume of the jump, and K value is calculated from equation 2.a.

$$K(\theta, F_1) = \bar{K}(\theta) - 0.05(F_1 - 3.5)$$

(2.a)

$\bar{K}(\theta)$ value is taken as 0.97 which is calculated from equation (Jan-Chang, 2009).

K and $L_r/(Y_2 - Y_1)$ vary through F_1 and G depends on F_1 and θ. Hence the modified sequent depth ratio is given by

$$\frac{Y_1}{Y_2} = \frac{1}{2}(\sqrt{1 + 8G_2^2} - 1)$$

(3)

Length of the Jump

To estimate the length of the jump analysis of energy dissipation is required for which length of the apron is to be measured. Length of the jump, Lj is the horizontal distance around the roller of jump or from the toe of jump to the section where the water surface becomes levelled after reaching the utmost depth. Hydraulic jump involves its length of jump L_j as the distance between two cross sections of the sequent depths Y_1 and Y_2. No bed protection is necessary for jump length beyond the downstream side of the hydraulic jump. The jump length is difficult to analyse during experiment because of hydraulic jump formation   and fluctuation of surface waves merged with turbulence.

Roller Length

The roller length (L_r) is the horizontal distance between the toes of the jump to the section where the flow depth reached a value of 98% of the tail water depth. This length is determined by visualization and with a float to localize the stagnation point.

The relationship between the roller length and sequent depth ratio is given as

$$\frac{L_r}{Y_1} = \frac{L_r}{Y_2 - Y_1}\left(\frac{Y_2}{Y_1} - 1\right)$$

(4)

Dimensionless Length

Dimensionless length of the jump is defined by ratio of length to the difference between the sequent depths. Dimensionless length is denoted as L_d.

$$L_d = 6.9(\eta - 1)\exp\left(-3.7\tan\theta\right)$$

(5)

Where η is the sequent depth ratio.

RESULTS AND DISCUSSION

Experimental investigations were conducted with different flow conditions ranging as follows. For Froude number ranging from 1.01 to 3.75, six slopes of the channel (2.22º, 5.41º, 8.62º, 11.86º, 15.31º, and 18.64º) were used. The discharges were varied accordingly with different slopes and gate openings to achieve the required range of inflow Froude number. Initial depth, sequent depth, discharge, average velocity and jump length were measured for the analysis.

The Important macroscopic parameters are initial depth (Y_1), sequent depth (Y_2), Initial mean velocity (V_1), mean velocity at the Table 1 presents the experimental investigation of hydraulic jump of previous researchers. In this study it is found that Froude number varies from 1.01 to 3.75 with the channel width 0.308 m, whereas the longitudinal distance of the toe jump from the upstream gate is 1.97 m to 4.07 m and the upstream flow depth ranges from 0.015 m to 0.303 m.

Table 1
Details of previous and present study experimental investigation of hydraulic jumps

Previous work	Width of channel(m)	Longitudinal distance of the jump toe from the upstream gate(m)	Upstream flow depth(m)	Froude number
Resh et al. (1974)	0.39	0.39-7.8	0.012-0.039	3.0-8.0
Babb & Aus (1981)	0.465	-	0.035	6.0
Long et al. (1991)	0.47	0.04-0.08	0.025	4-9
Liu et al. (2004)	0.46	0.1	0.041-0.071	2.0-3.3
Lennon & Hill (2006)	0.30	-	0.02-0.031	1.4-3.0
Valle & Pasternack (2006)	2	-	0.22	2.8
Chanson (2007)	0.25 and 0.50	0.50 and 1	0.013-0.029	5.1-8.6
Murzyn et al. (2007)	0.30	0.18-0.43	0.021-0.059	1.9-4.8
Wang & Chanson (2015)	0.50	0.80-1.87	0.012-0.047	3.8-8.5
Present study	0.308	1.97-4.07	0.015-0.030	1.01-3.75

Analysis of the Sequent Depth Ratio

Experimental runs were carried out to study the relationship among hydraulic jump, sequent depth ratio (η) and bed slope (S). Also the relationship between sequent depth ratio and Froude number were evaluated. The calculated and measured values for sequent depth ratio are plotted in Figures 3 and 4 against upstream Froude number (F_1).
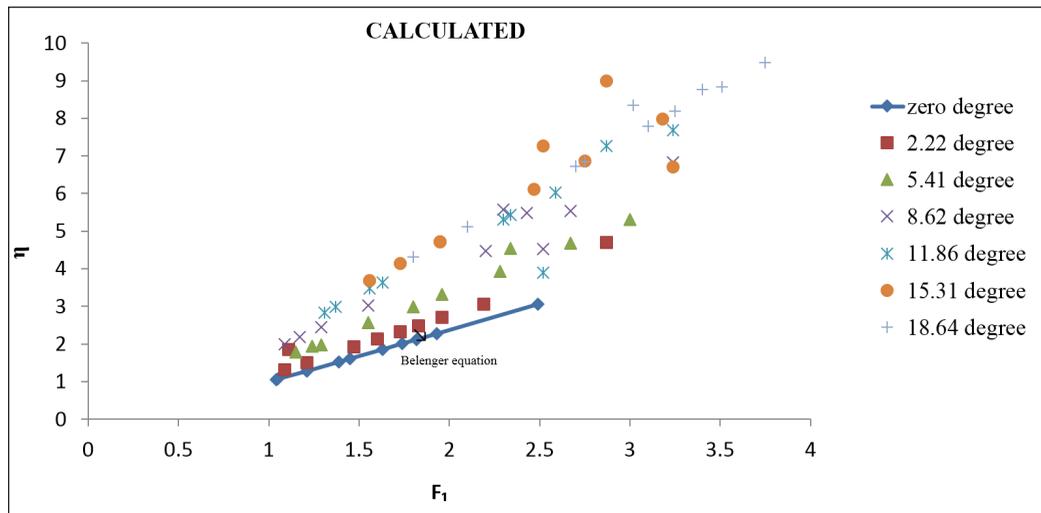

Figure 3. Comparison of variation of the calculated sequent depth ratio with upstream Froude number at different tilting angle
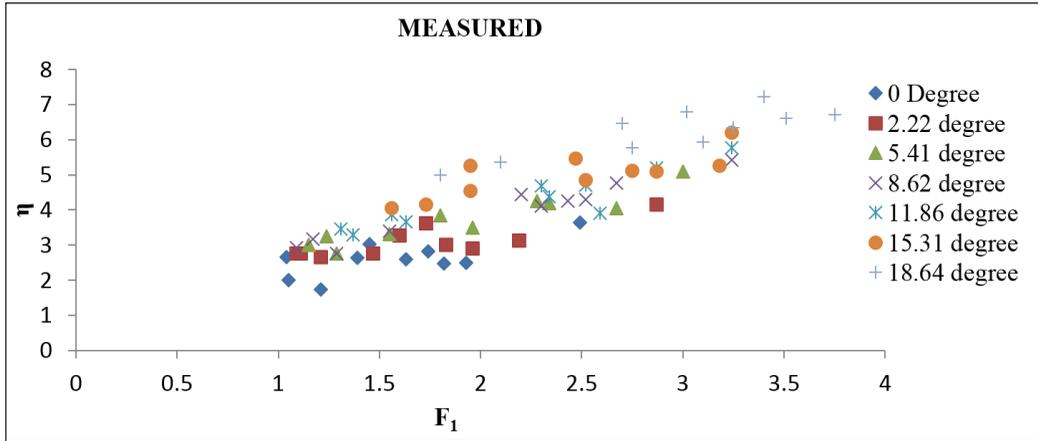
Figure 4. Comparison of variation of the calculated sequent depth ratio with Upstream Froude number at different tilting angle

Analysis of Hydraulic Jump Length

It is necessary to estimate the length of the jump through length of apron for energy dissipation. The length of the jump is the horizontal distance just about the roller of jump or from the toe of jump to a section where the water surface levels after reaching the maximum depth. The experimental runs were carried out to investigate the variation of the hydraulic jump with bed slope in present and past study made by Jan-Chang (2009) drawn in Figure 5. The present experimental data for ratio of roller length jump to pre hydraulic jump depth (L_r/Y_1) against sequent depth ratio (η), are plotted in Figure 6. The average values of L_d is evaluated as 2.50, 3.93, 4.86, 5.97 and 6.90 respectively and are presented in Table 3.
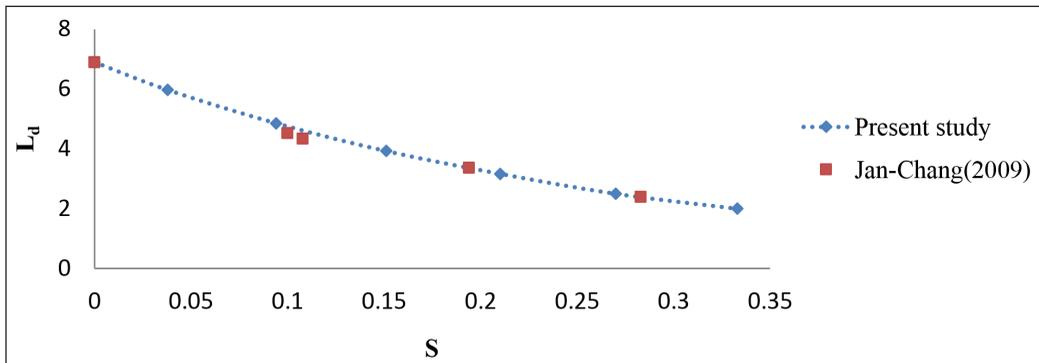

Figure 5. Comparison of relationship between dimensionless length of the Hydraulic Jump and flume bed slope
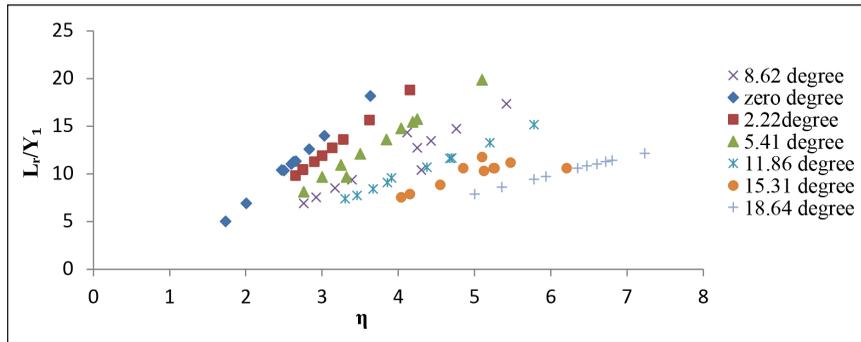
Figure 6. Comparison of ratio of Roller Length to pre hydraulic jump depth with Sequent Depth Ratio

Table 2
Experimental Results for Flows and computed hydraulic jump characteristics in the horizontal channel bed at different angles

Sl. No	Tilting angle	Mean measured roller length, L_r (m)	Mean Jump length, L_j (m)	Mean up-stream Froude number (F_1)	Mean calculated sequent depth ratio (η)	Mean calculated roller length, L_r (m)
1.	0°	0.2764	0.7728	1.575	1.783	0.3012
2.	2.22°	0.2996	1.1385	1.706	2.241	0.3231
3.	5.41°	0.3114	1.0238	1.928	3.304	0.3053
4.	8.62°	0.2591	0.261	0.9756	2.046	4.204
5.	11.86°	0.2284	0.2271	0.9605	2.173	4.854
6.	15.13°	0.2017	0.2012	1.2869	2.422	6.115
7.	18.46°	0.1937	0.1937	2.938	7.443	0.1835

Table 3
Calculation of dimensionless length of jump

Sl. no	Angle of flume bed (Degree)	Slope of channel Bed (S)	Dimensionless length of hydraulic Jump (L_d)
1.	0°	0.000	6.90
2.	2.22°	0.038	5.97
3.	5.41°	0.094	4.86
4.	8.62°	0.151	3.93
5.	11.86°	0.210	3.17
6.	15.31°	0.270	2.50
7.	18.46°	0.333	2.00

Dimensionless length of the jump is computed as ratio of hydraulic jump length to the difference between the sequent depths. From Figure 6, it is observed that as the bed slope increases the dimensionless length decreases and hence the height of jump (difference between post jump depth and pre-jump depth) decreases. The dimensionless length is highest with 6.9 at horizontal bed of the channel. As the bed slope increases and reaches

at 0.05, the L_d become 6 and decreases gradually up to bed slope 0.35. The value of L_d of tilting flume is lowest at slope18.46° and highest in horizontal bed of the flume at slope 0°. Based on the experimental result it is found that, the value of $L_{d\ is}$ considerable and dependent on tilting angle θ. The jump in a tilting flume of higher slope has little effect on length of jump.

Figure 7 represents the comparison of dimensionless jump length with Froude number at various tilting angle. The theoretical values of jump height for tilting angle ranging from 2.22º, 5.41º, 8.62º, 11.86º, 15.13º, and 18.46º are 5.97, 4.86, 3.93, 3.17, 2.5 and 1.98 respectively. The dimensionless jump length varies with variation of bed angles significantly and it is observed that 2.22 º maximum variations occur and Froude number varies abruptly.
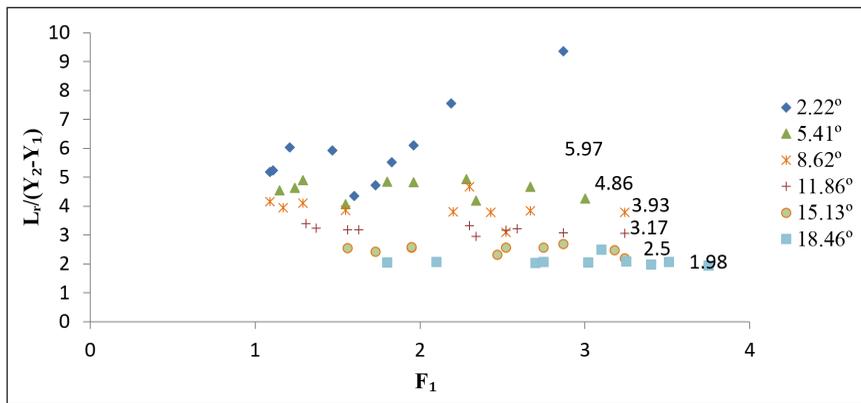

Figure 7. Dimensionless jump lengths versus approach Froude number at various flume bed angles

In this study the length of the jump means roller length of jump. In some instances of visual observation the jump was stabilized at upstream of the end sill. With a change in the Froude number, the roller length of the jump was also varied. This experimental runs were carried out to investigate the variation of the hydraulic jump length with the initial Froude's number as represented in figures. Froude number has linear relationship with L_r/Y_1. The variation of jump length in different bed slopes with Froude number at various tilting angle are presented in Figure 8.
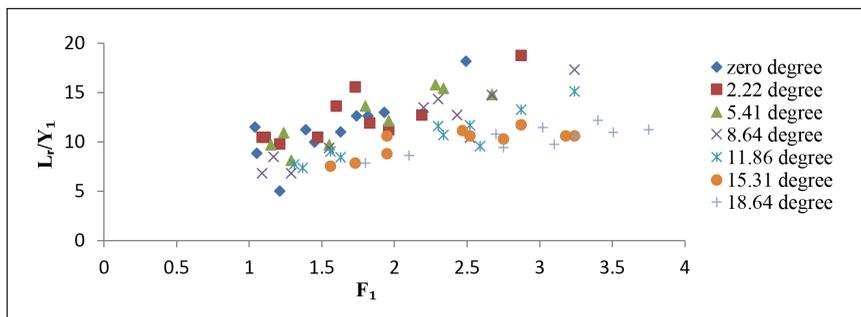

Figure 8. Comparison of variation of jump length in different bed slopes with Froude number

CONCLUSION

Design of hydraulic jump in stilling basins depends on combination of practical experience, theoretical analysis, and model studies. Laboratory tests are conducted here, to study the characteristics of hydraulic jump of a tilting flume of 4.90 meter long and 0.308 m breadth with Froude number ranges from 1.01 to 3.75. Experiments conducted in this study are to verify the theoretical equation for sequent depth ratio, roller length of the jump, and effective jump length. It is found that the values of the sequent depth ratio increases as the slope increases. At bed slope 18.64º, the sequent depth ratio is highest. The values of sequent depth ratio (η) for 2.22º, 5.41º, 8.62º, 11.86º, 15.13º and 18.46º are found to be limited within a range of 3.27% to 28.17%, 8.90% to 52%, 27.6% to 55%, 28.8% to 61.9%, and 44% to 70.6% respectively. Comparisons of the variation of Froude Numbers (calculated sequent depth ratio) and Froude Numbers (observed sequent depth ratio) with bed slope were made. The experimental measurement shows that as the bed slope increases, the Froude's number also increases with incremental variation of sequent depth ratios. The significance of result improves the design of hydraulic structures at laboratory condition to field condition. Variation characteristics of hydraulic jump will enable the designer to predict the required tail-water depth when the sequent depth ratio is known. Increasing the tail-water depth to the sequent depth will increase the stability of the jump. At bed slope 0º, 2.22º, 15.31º, the coefficient of determination and correlation coefficient differs much between calculated sequent depth ratio and measured sequent depth ratio. At bed slope 8.62º the calculated and measured value agrees closely and represents the best scenario for representing the perfect characteristics of hydraulic jump. The mean error for slope is 13.63% and the highest error is obtained for the slope of 2.22º in tilting flume. As the bed slope increases, the dimensionless length decreases and hence the height of jump decreases.

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